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## AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

### REPLY TO THE REPLIES TO MY "NOTE ON AVERAGE AND PROBABILITY."

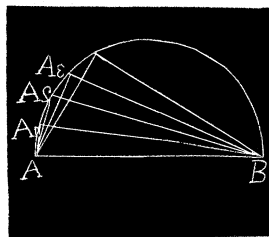
BY ARTEMAS MARTIN, LL. D., U. S. COAST AND GEODETIC SURVEY OFFICE,  
WASHINGTON, D. C.

I WISH to say first that I reaffirm all that I stated on pages 370 and 371, Vol. II., No. 12, and then proceed to consider the replies of the Repliers.

I. Professor Zerr starts out with the statement that "The problem that gives the result  $\frac{1}{4}a^2$  is different from the problem that gives the result  $\frac{a^2}{2\pi}$ ." This is superfluous information; I had clearly set forth that fact in my "Note." But the truth of the next sentence, "In the former the right angle remains fixed and does not lie on a circle as Dr. Martin states," I do not admit, and will proceed to prove its falsity.

Let  $AB=a$ , the given hypotenuse, which shall remain *fixed*.

Draw  $A_1B$ ,  $A_2B$ ,  $A_3B$ ,  $A_4B$ , and so on, the sides  $AA_1$ ,  $AA_2$ ,  $AA_3$ ,  $AA_4$ , etc., increasing uniformly from 0 towards  $a$ , the consecutive differences,  $AA_2-AA_1$ ,  $AA_3-AA_2$ ,  $AA_4-AA_3$ , etc., being all equal to each other, and each difference less than any assignable quantity. Thus will be had *all possible* right-angled triangles having the hypotenuse  $a$ , and, as I stated on page 371, the right angles *are* all situated on the semi-circumference whose diameter is the given hypotenuse  $a$ ; but they (the right angles) are not uniformly distributed on this semicircumference because the chords  $AA_1$ ,  $AA_2$ ,  $AA_3$ ,  $AA_4$ , etc., increase (or vary) uniformly and therefore their arcs can not increase (or vary) uniformly.



Professor Zerr continues: "The problem [the one that gives the result  $\frac{1}{4}a^2$ ] is as follows: 'Find the average area of all triangles formed by a straight line of constant length  $a$  sliding so that its extremities constantly touch two fixed straight lines at right angles to one another'." With all due deference to Professor Zerr, I beg leave to say that I have *not* conceived the triangles to be generated in any such way, as I have clearly shown by the diagram above.

The remainder of Professor Zerr's "Note" does not require considering as it has nothing to do with the matter in hand.

II. I discard the "tail" in italics Professor Matz has appended to the problem; it is not needed to "fly the kite."

I will take up his third and fourth paragraphs. In his third paragraph he says that I, by making the number of possible right-angled triangles "proportional to the given hypotenuse," *ignore* an infinitude of right-angled tri-

angles. Now if Professor Matz can *prove* that there *are any* right-angled triangles having the hypotenuse  $a$  besides those obtained by varying one leg uniformly from 0 to  $a$ , I—would like to see the proof. How *can* there *be* any other triangles, if we have a leg for *every possible* value from 0 to  $a$ ?

III. I will pass over the first and second paragraphs of the Editor's "Reply." In regard to the third paragraph I deny that any triangles *can* be interpolated, and demand proof. If one leg takes *all possible* values from 0 to  $a$ , every triangle has been included and there *can not* be any other.

IV. My solution, which I desire to reproduce here, is as follows:

Let  $x$  denote one leg of any one of the triangles, then  $\sqrt{a^2 - x^2}$  will denote the other leg. The area of this triangle is  $\frac{1}{2}x\sqrt{a^2 - x^2}$ , and the true average of this is

$$\int_0^a \frac{1}{2}x\sqrt{a^2 - x^2} \div \int_0^a dx = \frac{1}{6}a^2.$$

V. I think I have considered and fully refuted every objection that has been raised against my solution.

*Correction.*—Vol. II., page 371, for "p. 82" read p. 282.

## MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

## SOLUTIONS OF PROBLEMS.

35. Proposed by WILLIAM SYMMONDS, A. M., Professor of Mathematics and Astronomy in Pacific College, Santa Rosa, California; P. O., Sebastopol, California.

To an observer whose latitude is 40 degrees north, what is the sidereal time when Fomalhout and Antares have the same altitude; taking the Right Ascension and Declination of the former to be 22 hours, 52 minutes, —30 degrees, 12 minutes; of the latter 16 hours, 23 minutes, —26 degrees, 12 minutes?

II. Corrected solution by JOHN M. ARNOLD, Crompton, Rhode Island; and Prof. G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let  $\lambda$ =latitude of observer,  $\alpha$ ,  $\delta$ ,  $\alpha_1$ ,  $\delta_1$  the Right Ascension and Declination of Fomalhout and Antares, respectively,  $\beta$ =altitude,  $h$ ,  $h_1$  the hour angles.

This event can happen only when Antares is west and Fomalhout east of the meridian.

$$\therefore \left. \begin{aligned} \sin \beta &= \sin \lambda \sin \delta + \cos \lambda \cos \delta \cos h \\ &= \sin \lambda \sin \delta_1 + \cos \lambda \cos \delta_1 \cos h_1 \end{aligned} \right\} \dots \dots \dots (1).$$